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Operations on Dutch Windmill Graph of Topological indices

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Abstract

Topological indices are well studied in recent years. These are useful tools in studying Quantitative Structure Activity Relationship (QSAR) and Quantitative Structure Property Relationship (QSPR). The main goal of this paper is to concentrate the investigation on generalized version of Dutch windmill graph of certain graph operators in terms of topological indices, for instance, symmetric division deg index, first and second Zagreb indices.

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1 Introduction and Preliminaries

A systematic study of topological indices is one of the most conspicuous aspects in many branches of mathematics. In [7], discrete mathematical chemistry and topological indices have provided evidence to be helpful in the study of QSPR (Quantitative Structure Property Relationships) models. There are several prominent indices applied in chemical engineering (for instance, QSPR/QSAR study) for grasping the relationship between the molecular structure and their potential physico-chemical characteristics such as Symmetric Division Deg *SDD* index [2, 4, 5], Zagreb indices.

Topological index is a type of molecular descriptor that is calculated based on a molecular graph of a chemical compound. Most of the proposed topological indices are related to either vertex adjacency of a graph or graph distances in the graph. One of the most widely known topological indices is Wiener index named after chemist Harold Wiener. There are some topological indices based on degrees such as the Randic index, Reciprocal Randic index, Zagreb index, Harmonic index, *ABC* index etc.

Among the 148 discrete Adriatic indices studied in [14], whose predictive properties were evaluated against the benchmark data sets of the International Academy of Mathematical Chemistry [13], few indices were selected as significant predictors of physicochemical properties. In fact *SDD* index is one of the discrete adriatic indices that is good predictor of total surface area for polychlorobiphenyls.

In the next two paragraphs, we recall some definitions which will be needed for our theory.

While the *first Zagreb index* [9] is defined as

$$M_1(G) = \sum d(u)^2 = \sum_{u,v \in E(G)} [d(u) + d(v)], \tag{1.1}$$

the *general second Zagreb index* [10] is defined as

$$M_2(G) = \sum_{u,v \in E(G)} [d(u)d(v)] \tag{1.2}$$

On the other hand, the *symmetric division deg SDD* index [4] is one of the discrete Adriatic indices that is good predictor of total surface area for polychlorobiphenyls. The SDD index of a connected graph G is defined as

$$\begin{aligned} SDD(G) &= \sum_{uv\in E(G)} \frac{max(d_u,d_v)}{min(d_u,d_v)} + \frac{min(d_u,d_v)}{max(d_u,d_v)} \\ &= \sum_{uv\in E(G)} \frac{d_u}{d_v} + \frac{d_v}{d_u} \\ &= \sum_{uv\in E(G)} \frac{{d_u}^2 + {d_v}^2}{d_u d_v}, \end{aligned} \quad (1.3)$$

where d_v is the degree of a vertex v in G.

According to the famous book [6], while the *semi total (line) graph* $T_1(G)$ of G is the graph whose point set is $V(G)\bigcup X(G)$ where two points are adjacent if and only if

(i) they are adjacent lines of G, or

(ii) one is a point of G and another is a line of G incident with it,

the *semi total (point) graph* $T_2(G)$ of G is the graph whose point set is $V(G)\bigcup X(G)$ where two point are adjacent if and only if

(i) they are adjacent point of G, or

(ii) one is a point of G and another is a line of G incident with it.

Finally, again by [6], the *total graph* $T(G)$ of graph G is the graph whose vertex set $V\bigcup E$ with two vertices of $T(G)$ being adjacent if and only if the corresponding elements of G are adjacent or incident.

In this article, we establish the SDD index, First and Second Zagreb indices of the Dutch windmill graph of semi total (point and line) and total graphs.

2 Main Results

In this section we presents SDD and Zagreb indices over different formed of Dutch windmill graph.

Theorem 1 *The symmetric division deg index of Dutch windmill graphs is*

$$SDD(D_n^m) = 2(m-1)^2 + 2mn\,.$$

Proof. Dutch windmill graph D_n^m contains $(n-2)m$ vertices of degree two and one vertex of degree $2m$. We partition the edges of D_n^minto edges of the type $E_{(d_u,d_v)}$ where uv is an edge. In D_n^m, we get edges of the type $E_{(2,2)}$ and $E_{(2m,2)}$. The number of edges of these types are given in the Table 1. Edge partition based on degrees of end vertices of each edge.

$E_{(d_u,d_v)}$	$E_{(2,2)}$	$E_{(2m,2)}$
No. of edges	$m(n-2)$	$2m$

Table 1: Edge partition of Dutch wind mill graph

Now, by replacing suitable arrangements in (1.3), we obtain

$$SDD(D_n^m) =\mid E_{(2,2)} \mid \sum_{uv\in E_{(2,2)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} + \mid E_{(2m,2)} \mid \sum_{uv\in E_{(2m,2)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v}$$

$$\begin{aligned}
&= m(n-2)\left[\frac{2^2+2^2}{2.2}\right] + 2m\left[\frac{2^2+(2m)^2}{2.2m}\right] \\
&= 2m(n-2) + (2m)^2 + 2 \\
&= 2(m-1)^2 + 2mn\,,
\end{aligned}$$

as required. ♢

Theorem 2 *The SDD index of a semi total (line) graph formed by Dutch windmill graph is*

$$SDD(T_1(D_n^m)) = 7mn + 0.8m^2 - 2.1m + 5\,.$$

Proof. Let us split the edges of the type $E_{(d_u,d_v)}$ where uv is an edge in the Dutch windmill graph D_n^m. In semi total (line)graph of D_n^m, we get edges of the type $E_{(5,5)}$, $E_{(2,5)}$, $E_{(2,4)}$, $E_{(2m,5)}$, $E_{(4,4)}$ and $E_{(5,4)}$. The number of edges of these types are given in the Table 2.

$E_(d_u, d_v)$	$E_{(5,5)}$	$E_{(2,5)}$	$E_{(2,4)}$	$E_{(2m,5)}$	$E_{(4,4)}$	$E_{(5,4)}$
No. of edges	$2m$	$2m$	$2m(n-2)$	$2m$	$mn-3m$	$2m$

Table 2: Edge partition of semi total (line) graph formed by Dutch windmill graph.

Now, by replacing suitable arrangements in (1.3), we obtain

$$SDD(T_1(D_n^m)) = \mid E_{(5,5)} \mid \sum_{uv \in E_{(5,5)}(N)} \frac{d_u^2 + d_v^2}{d_u.d_v} + \mid E_{(2,5)} \mid \sum_{uv \in E_{(2,5)}(D_n^m)} \frac{d_u^2 + d_v^2}{d_u.d_v}$$
$$+ \mid E_{(2,4)} \mid \sum_{uv \in E_{(2,4)}(D_n^m)} \frac{d_u^2 + d_v^2}{d_u.d_v} + \mid E_{(2m,5)} \mid \sum_{uv \in E_{(2m,5)}(D_n^m)} \frac{d_u^2 + d_v^2}{d_u.d_v}$$
$$+ \mid E_{(4,4)} \mid \sum_{uv \in E_{(4,4)}(D_n^m)} \frac{d_u^2 + d_v^2}{d_u.d_v} + \mid E_{(5,4)} \mid \sum_{uv \in E_{(5,4)}(D_n^m)} \frac{d_u^2 + d_v^2}{d_u.d_v}$$

$$= 2m\left[\frac{5^2+5^2}{5.5}\right] + 2m\left[\frac{2^2+5^2}{2.5}\right] + 2m(n-2)\left[\frac{2^2+4^2}{2.4}\right]$$
$$+ 2m\left[\frac{(2m)^2+5^2}{2m.5}\right] + (mn-3m)\left[\frac{4^2+4^2}{4.4}\right] + 2m\left[\frac{5^2+4^2}{5.4}\right]$$

$$= 7mn + 0.8m^2 - 2.1m + 5\,.$$

Hence the result. ♢

Theorem 3 *The SDD index of a semi total (point) graph formed by Dutch windmill graph is*

$$SDD[T_2(D_n^m)] = 7mn + 3(m-1)(2m-1)\,.$$

Proof. Let us consider partition the edges of D_n^m into edges of the type $E_{(d_u,d_v)}$, where uv is an edge. In the semi total (point) graph of D_n^m, we have edges of the type $E_{(4m,2)}$, $E_{(4m,4)}$, $E_{(2,4)}$ and $E_{(4,4)}$. In fact the number of edges of these types are given in the Table 3.

$E_{(d_u,d_v)}$	$E_{(4m,2)}$	$E_{(4m,4)}$	$E_{(2,4)}$	$E_{(4,4)}$
No. of edges	$2m$	$2m$	$m(n-2)$	$2m(n-1)$

Table 3: Edge partition of semi total (point) graph formed by Dutch windmill graph.

By arrangements in (1.3), we get

$$\begin{aligned} SDD(T_2(D_n^m)) &= \mid E_{(4m,2)} \mid \sum_{uv\in E_{(4m,2)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} + \mid E_{(4m,4)} \mid \sum_{uv\in E_{(4m,4)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} \\ &+ \mid E_{(4,4)} \mid \sum_{uv\in E_{(4,4)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} + \mid E_{(2,4)} \mid \sum_{uv\in E_{(2,4)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} \\ &= 2m\left[\frac{4m^2+2^2}{4m.2}\right] + 2m\left[\frac{4m^2+4^2}{4m.4}\right] + m(n-2)\left[\frac{4^2+4^2}{4.4}\right] \\ &+ 2m(n-1)\left[\frac{2^2+4^2}{2.4}\right] \\ &= 7mn + 3(m-1)(2m-1)\,, \end{aligned}$$

as required. $\diamondsuit$

Theorem 4 *Let $T(D_n^m)$ be the total graph of Dutch windmill graph. Then SDD index of $T(D_n^m)$ is*

$$11mn + 5.6m^2 - 6.8m + 3.5\,.$$

Proof. When we consider the Dutch windmill graph D_n^m, we get edges of the type $E_{(2,4)}$, $E_{(4m,5)}$, $E_{(4m,2)}$, $E_{(5,4)}$, $E_{(5,5)}$ and $E_{(4,4)}$ in the total graph of D_n^m. The number of edges of these types are given in the Table 4.

$E_{(d_u,d_v)}$	$E_{(2,4)}$	$E_{(4m,5)}$	$E_{(4m,2)}$	$E_{(5,4)}$	$E_{(5,5)}$	$E_{(4,4)}$
No. of edges	$2m(n-1)$	$2m$	$2m$	$4m$	$2m$	$3mn-7m$

Table 4: Edge partition of total graph of Dutch windmill graph.

Considering (1.3), we have

$$
\begin{aligned}
SDD(T(D_n^m)) =& \mid E_{(2,4)} \mid \sum_{uv\in E_{(2,4)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} + \mid E_{(4m,5)} \mid \sum_{uv\in E_{(4m,5)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} \\
&+ \mid E_{(4m,2)} \mid \sum_{uv\in E_{(4m,2)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} + \mid E_{(5,4)} \mid \sum_{uv\in E_{(5,4)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} \\
&+ \mid E_{(5,5)} \mid \sum_{uv\in E_{(5,5)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v} + \mid E_{(4,4)} \mid \sum_{uv\in E_{(4,4)}(D_n^m)} \frac{d_u^2+d_v^2}{d_u.d_v}
\end{aligned}
$$

$$
= 2m(n-1)\left[\frac{2^2+4^2}{2.4}\right] + 2m\left[\frac{(4m)^2+5^2}{4m.5}\right] + 2m\left[\frac{(4m)^2+2^2}{4m.2}\right] + 4m\left[\frac{(5)^2+4^2}{5.4}\right]
$$
$$
+ 2m\left[\frac{5^2+5^2}{5.5}\right] + (3mn-7m)\left[\frac{4^2+4^2}{4.4}\right]
$$

$$
= 11mn + 5.6m^2 - 6.8m + 3.5\,.
$$

This gives the result. $\diamondsuit$

The proofs of the following three results can be obtained easily by taking into account the equations given in (1.1) and (1.2).

Corollary 1 *Let $M_1(D_n^m)$ and $M_2(D_n^m)$ be the first and second Zagreb indices, respectively, of a Dutch windmill graph. Then*

$$4m^2 - 4m + 4mn \quad \textit{and} \quad 8m^2 - 8m + 4mn\,.$$

Corollary 2 *Let $M_1(T_1(D_n^m))$ and $M_2(T_1(D_n^m))$ be the first and second Zagreb indices, respectively, of a Dutch windmill graph of semi total (point) graph. Then*

$$16m^2 - 16m + 20mn \quad \textit{and} \quad 48m^2 + 48m + 32mn\,.$$

Corollary 3 *Let $M_1(T_2(D_n^m))$ and $M_2(T_2((D_n^m))$ be the first and second Zagreb indices, respectively, of a Dutch windmill graph of semi total (line) graph. Then*

$$4m^2 + 14m + 20mn \quad \textit{and} \quad 20m^2 + 30m + 32mn\,.$$

The next corollary shows that one can get similar results for a graph other than the Dutch windmill graph.

Corollary 4 *Let F_m^n be the friendship graph. Then we have the following.*

- $SDD[F_m^n] = 2m^2 + 2m + 2\,.$
- $SDD[T_1(F_m^n)] = 0.8m^2 + 18.9m + 5\,.$
- $SDD[T_2(F_m^n)] = 6m^2 + 12m + 3\,.$
- $SDD[T(F_m^n)] = 5.6m^2 + 26.2m + 3.5\,.$

Conclusion 1 *In this paper, by considering semi-total line graphs, semi-total point graphs and total graph operators, we computed certain topological indices for a generalized Dutch windmill graph which gives a new direction in the field of structural chemistry.*

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